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Assignment on: Signal & System

1. **What are the different criteria for system stability?**

**answer:**

**Different Criteria for System Stability**

Stability is a fundamental property of a system that determines whether its output remains under control when subjected to an input. A system is considered **stable** if its output does not diverge uncontrollably in response to bounded inputs or initial conditions.

There are several criteria used to analyze system stability, particularly in **control systems, signal processing, and system analysis**. The key stability criteria are discussed below:

**1. Bounded Input Bounded Output (BIBO) Stability**

A system is said to be **BIBO stable** if every bounded input results in a **bounded output**. This means that if the input signal remains finite, the output should also remain finite.

**Mathematical Condition:**

For a **continuous-time** system with **impulse response** h(t)h(t), BIBO stability is defined as:

For a **discrete-time** system, the condition is:

**Explanation:**

* The impulse response h(t) represents the system’s reaction to a unit impulse input.
* If h(t) is **absolutely integrable**, then the system is BIBO stable.
* If this condition is violated, the system may produce an **unbounded** (exploding) output even for a bounded input.

**Example:**

* Consider the system with impulse response:
* Since , the system is stable.
* However, the integral diverges, making the system **unstable**.

**2. Asymptotic Stability**

A system is **asymptotically stable** if its **natural response** (i.e., the response due to initial conditions without external input) **decays to zero** over time.

**Conditions:**

For a **continuous-time system**, the system is asymptotically stable if all **poles of the system’s transfer function** H(s) have **negative real parts**:

Re() < 0 ∀i

For a **discrete-time system**, the system is asymptotically stable if all **poles** are **inside the unit circle** in the **z-plane**:

∣zi∣ < 1 ∀i

**Example:**

Consider the system with transfer function:

H(s) *=*

Since the pole is at s=−3 (negative real part), the system is **asymptotically stable**.

However, if the transfer function was:

H(s) *=*

The pole is at s=3 (positive real part), leading to an **unstable system**.

**3. Marginal Stability**

A system is **marginally stable** if its natural response neither grows unbounded nor decays to zero but **remains constant** or oscillatory over time.

**Conditions:**

* In **continuous-time systems**, marginal stability occurs if there are **poles on the imaginary axis** (jω) but **none in the right half-plane**.
* In **discrete-time systems**, the system is marginally stable if it has **poles on the unit circle** but **none outside**.

**Example:**

The system with transfer function:

H(s) *=*

has poles at s=±j (purely imaginary). The response is **oscillatory** and does not decay, making the system **marginally stable**.

However, if a small disturbance is introduced, the system may become **unstable**.

**4. Lyapunov Stability**

This concept is widely used in **control theory**, particularly for **nonlinear systems**. A system is **Lyapunov stable** if, when subjected to small disturbances, it remains close to its equilibrium position.

**Mathematical Definition:**

Let V(x) be a **Lyapunov function** (like an energy function). If:

1. V(x) > 0 (positive definite)
2. V˙(x)≤ 0 (non-increasing)

Then, the system is **stable**.

**Significance:**

* Lyapunov stability is useful when dealing with **complex nonlinear systems** where eigenvalues and poles are not easily calculated.
* It helps determine **whether a small perturbation will cause the system to return to equilibrium or diverge**.

**5. Routh-Hurwitz Stability Criterion (for Continuous Systems)**

This criterion determines whether a **continuous-time system** is stable **without solving for the roots** of its characteristic equation.

**Steps:**

1. Write the characteristic equation: + + … + + = 0
2. Construct the **Routh array**.
3. Check the **first column**:
   * If all elements are **positive**, the system is **stable**.
   * If there are sign changes, the system has **unstable poles**.

**Example:**

For the equation:

If the **Routh array** has all positive values in the first column, the system is **stable**.

**6. Jury’s Stability Criterion (for Discrete Systems)**

Jury’s criterion is used for **discrete-time systems**, ensuring that all poles are **inside the unit circle**.

**Steps:**

1. Construct the **Jury table** from the characteristic equation.
2. Check sign conditions for stability.

**Example:**

For the discrete system:

Jury’s test helps determine if the roots lie inside the **unit circle**, ensuring stability.

**Conclusion**

System stability is crucial in engineering, ensuring that real-world systems like **control systems, electronic circuits, and signal processors** function correctly. The choice of stability criterion depends on whether the system is **continuous or discrete**, **linear or nonlinear**, and whether **frequency domain or time domain analysis** is preferred.

**Summary Table:**

| **Stability Type** | **Condition for Stability** |
| --- | --- |
| **BIBO Stability** | ( \int |
| **Asymptotic Stability** | All poles in **left-half s-plane** or **inside the unit circle** |
| **Marginal Stability** | Poles on **jω-axis (continuous)** or **unit circle (discrete)** |
| **Lyapunov Stability** | V(x)>0, V˙(x)≤0 |
| **Routh-Hurwitz Criterion** | No sign changes in Routh array |
| **Jury’s Stability Criterion** | All conditions in Jury’s test satisfied |

This comprehensive discussion ensures a deep understanding of system stability in different contexts.

**2.What are the basic properties of a system? Explain causality and stability.**

**answer:**

**Basic Properties of a System & Explanation of Causality and Stability**

A system is a mathematical model that describes how input signals are processed to produce output signals. Systems have several fundamental properties that help in analyzing their behavior.

**Basic Properties of a System**

**1. Linearity**

A system is **linear** if it follows the principles of **superposition** and **homogeneity**.

* **Superposition:** If input (t) produces output and produces then for a combined input: ***(t)***
* The output should be: ***(t)***
* **Example:** A resistor in an electrical circuit follows Ohm’s Law, making it linear.

**2. Time-Invariance**

A system is **time-invariant** if a time shift in the input results in the same time shift in the output.

* Mathematically, if input produces output then for a delayed input the output should be  **.**
* **Example:** A system that multiplies an input signal by is time-invariant, but a system where is time-variant.

**3. Causality**

A system is **causal** if its output at any time **depends only on past and present inputs**, not future inputs.

* **Mathematical Condition:**
* **Example:** A real-world system like an audio amplifier is causal, as it only responds to sounds **after** they occur. However, a system requiring future values of input is **non-causal**.

**4. Stability**

A system is **stable** if a bounded input always produces a bounded output (**BIBO Stability**).

**Mathematical Condition:**

* **Example:** A speaker system with controlled volume is stable, while an uncontrolled microphone feedback loop can become unstable.

**5. Memory**

A system has **memory** if its output depends on past inputs; otherwise, it is **memoryless**.

* **Example:** A resistor (Ohm’s Law: V=IR) is memoryless, but a capacitor (which depends on past voltage values) has memory.

**6. Invertibility**

A system is **invertible** if its input can be uniquely determined from its output.

* **Example:** A simple multiplication system is invertible, but a system that squares the input is not, since both and give the same output.

**Explanation of Causality and Stability**

**Causality:**

* A system is causal if it **does not depend on future inputs**.
* All **real-time physical systems** are causal.
* **Example:** A live television broadcast is causal because it only transmits events **as they happen**.

**Stability:**

* Stability ensures that a system's output **does not grow indefinitely** for a limited input.
* Unstable systems can **explode** or behave unpredictably.
* **Example:** A financial system where a small investment results in uncontrolled profits or losses is unstable.

**Conclusion**

Understanding system properties helps in designing and analyzing electrical, mechanical, and control systems effectively. **Causality ensures real-time response, while stability ensures predictable behavior.**

1. **Define and explain the significance of the impulse response of an LTI system.**

**answer:**

### ****Impulse Response of an LTI System: Definition & Significance****

#### ****Definition:****

The **impulse response** of a **Linear Time-Invariant (LTI) system** is its output when the input is a **unit impulse function** δ(t)\delta(t) (in continuous-time) or δ[n]\delta[n] (in discrete-time).

* For a **continuous-time LTI system**, the impulse response is denoted as **h(t)h(t)**,

where:

(Convolution integral)

* For a **discrete-time LTI system**, it is denoted as **h[n]**,

where:

(Convolution sum)

#### ****Significance of Impulse Response:****

1. **Characterizes the System Completely:**
   * The impulse response provides a **complete description** of an LTI system’s behavior.
   * Once (or ) is known, the system's output for any input can be computed using **convolution**
2. **Determines Stability:**
   * A system is **BIBO stable** if its impulse response is **absolutely integrable**:

For discrete-time systems:

* + If the impulse response grows indefinitely, the system is **unstable**.

1. **Determines Causality:**
   * A system is **causal** if its impulse response is **zero for t<0 (or n<0 in discrete-time)**:
   * Meaning, the system does not depend on **future** inputs.
2. **Useful in System Analysis (Fourier & Laplace Transforms):**
   * The **Fourier Transform** of the impulse response gives the system’s **frequency response**:
   * The **Laplace Transform** of h(t)h(t) gives the system’s **transfer function**:
3. **Impulse Response in Real-World Applications:**
   * In **audio systems**, the impulse response helps design **equalizers** and **reverberation effects**.
   * In **control systems**, it is used to design stable feedback loops.
   * In **communication systems**, it helps model the behavior of transmission channels.

### ****Conclusion:****

The impulse response is a **fundamental tool** in analyzing LTI systems. It helps predict system behavior, check stability and causality, and is widely used in engineering applications.

1. **State and proof Parseval’s theorem.**

**answer:**

### ****Statement and Proof of Parseval’s Theorem****

#### ****Statement of Parseval’s Theorem:****

Parseval’s theorem states that the total **energy** of a signal in the **time domain** is equal to the total **energy** in the **frequency domain**.

For a **continuous-time** signal x(t) , Parseval’s theorem is given by:

where:

* is the signal in the time domain.
* is the Fourier Transform of .

For a **discrete-time** signal x[n], Parseval’s theorem is:

where is the **Discrete-Time Fourier Transform (DTFT)** of .

**Proof of Parseval’s Theorem (Continuous-Time Case)**

1. **Fourier Transform Definition:**

The Fourier Transform of is:

and its inverse is: =

1. **Energy of the Signal in Time Domain:**  
   The **total energy** of the signal is given by: E
2. **Substituting x(t) from its Inverse Fourier Transform:**

Using the inverse Fourier transform of x(t) , we get: =

and its complex conjugate: =

1. **Computing the Energy Integral:**

Substituting and :

1. **Rearrange the Integrals:**

Interchanging the order of integration:

The inner integral evaluates to the **Dirac delta function**

1. **Final Step:**

Since we obtain

This proves Parseval’s theorem.

**Significance of Parseval’s Theorem:**

**Energy Conservation:** It shows that the total energy remains the same in both time domain and frequency domain.

**Practical Applications**: Used in signal processing, communications, and engineering to analyze power and energy distributions in different domains.

**Fourier Transform Interpretation:** Helps in understanding how signals behave across different frequency components.

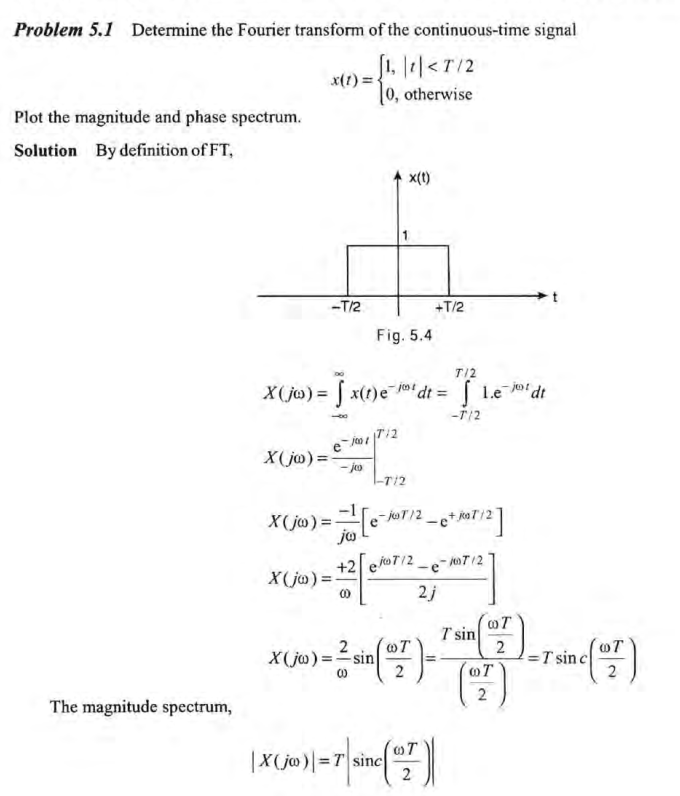
**Conclusion:**

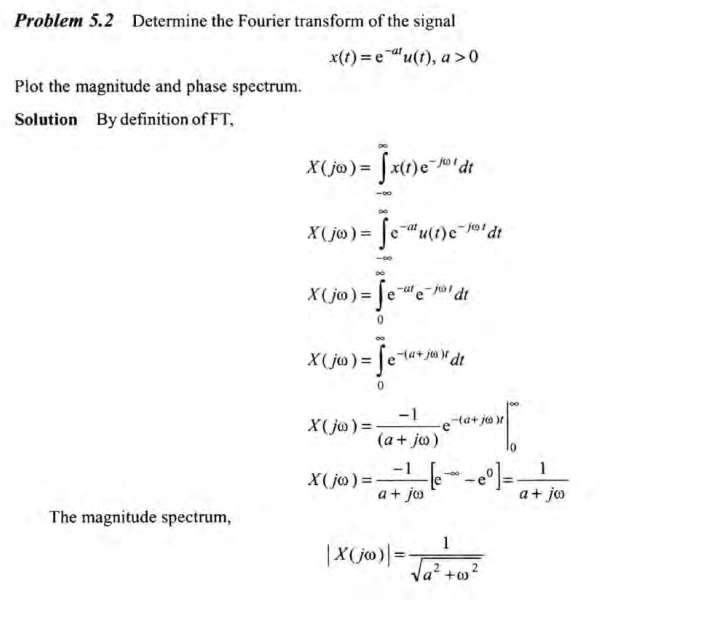
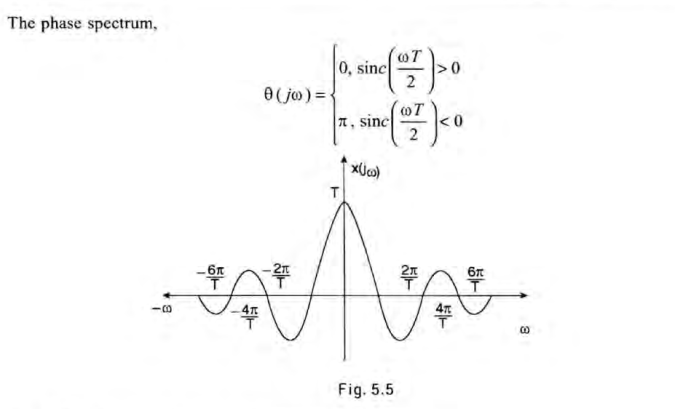
Parseval’s theorem is a fundamental result in Fourier analysis that ensures energy equivalence between time and frequency domains, playing a crucial role in signal processing, electrical engineering, and physics

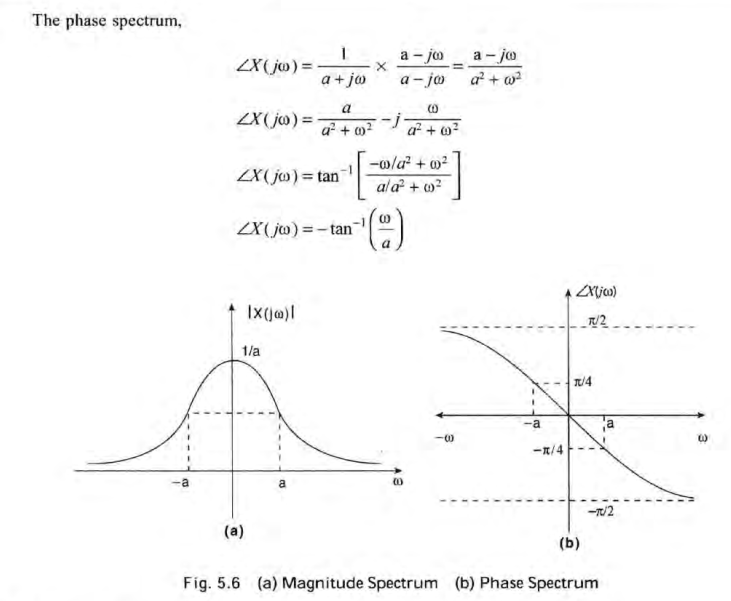
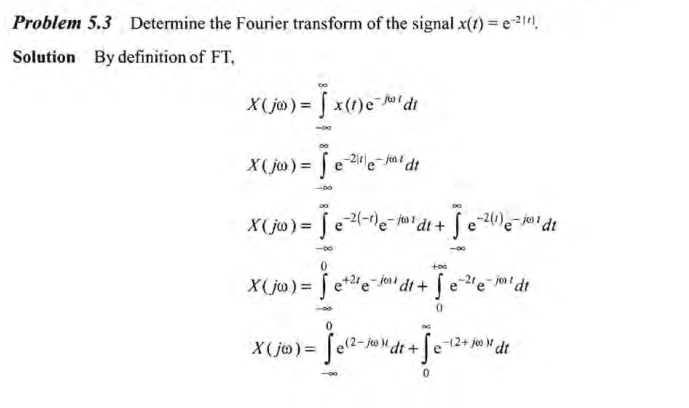
1. **Math of Fourier Transform.**

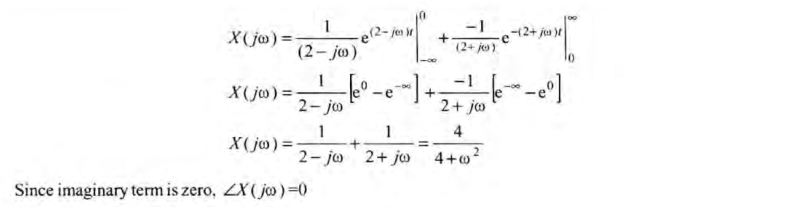
**answer:**

Here are some important maths of Fourier Transform:



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